You wish to estimate $\int_1^{10} \ln x \, dx$ using numerical approximation. **SCORE: _____ / 12 PTS**

[a] Use fnInt to find the value of $\int_1^{10} \ln x \, dx$.

14.02585093 (1)

[b] Find S_6 **WITHOUT using the INTEGRAL program demonstrated in lecture.**

Show the calculation (which should involve the *ln* function) that gave your answer.

$$\Delta x = \frac{10-1}{6} = \frac{3}{2}$$

$$S_6 = \frac{1}{3} \cdot \frac{3}{2} \left(\ln 1 + 4 \ln \frac{5}{2} + 2 \ln 4 + 4 \ln \frac{11}{2} + 2 \ln 7 + 4 \ln \frac{17}{2} + \ln 10 \right) \; (3)$$

$$\approx 14.00570703 \; (1)$$

[c] Find the value of ES_6. Round your answer to 4 significant digits (ie. 4 decimal places starting at the first non-zero digit).

$$\int_1^{10} \ln x \, dx - S_6 = 0.02014 \; (1)$$

[d] Using the error bound formulae, find the minimum number of subintervals n necessary so that your approximation is within 10^{-4} of the actual integral if you use

[i] trapezoidal rule

$$|(\ln x)''| = |-x^{-2}| \le 1 \text{ on } [1, 10]$$

$$\frac{1(10-1)^3}{12n^2} \le 10^{-4} \; (2)$$

$$\frac{10^4 \cdot 9^3}{12} \le n^2$$

$$n \ge \sqrt{\frac{10^4 \cdot 9^3}{12}} \approx 779.4$$

MINIMUM $n = 780$ (1)

[ii] Simpson's rule

$$|(\ln x)^{(4)}| = |-6x^{-4}| \le 6 \text{ on } [1, 10]$$

$$\frac{6(10-1)^5}{180n^4} \le 10^{-4} \; (2)$$

$$\frac{6 \cdot 10^4 \cdot 9^5}{180} \le n^4$$

$$n \ge \sqrt[4]{\frac{6 \cdot 10^4 \cdot 9^5}{180}} \approx 66.6$$

MINIMUM $n = 68$ (1)

Use the table of integrals on the instructor's website to evaluate the following integrals. **SCORE: _____ / 8 PTS**
State clearly the rule numbers of all rules you used, and state the values for each of the parameters in those rules (ie. $a, b, n \ldots$).

[a] $\displaystyle\int \frac{\sec 2x \tan 2x}{\sqrt{3\cos 2x - 5}}\,dx$ 1 $u = \cos 2x$

$du = -2\sin 2x\,dx$

$\dfrac{\sec 2x \tan 2x}{\sqrt{3\cos 2x - 5}} \; \dfrac{du}{-2\sin 2x}$

1

$= \dfrac{-\frac{1}{2}du}{\cos^2 2x\sqrt{3\cos 2x - 5}} = -\dfrac{1}{2}\dfrac{du}{u^2\sqrt{3u-5}}$

$\frac{1}{2}$ 62 $n=2,\ a=-5,\ b=3$

1 $-\dfrac{1}{2}\left(-\dfrac{\sqrt{3u-5}}{-5(1)u^1} - \dfrac{3(1)}{2(-5)(1)}\displaystyle\int \frac{du}{u\sqrt{3u-5}}\right)$ 57 $a=-5,\ b=3$ $\frac{1}{2}$

$= -\dfrac{\sqrt{3u-5}}{10u} - \dfrac{3}{20}\left(\dfrac{2}{\sqrt{5}}\tan^{-1}\sqrt{\dfrac{3u-5}{5}}\right) + C = -\dfrac{\sqrt{3\cos 2x - 5}}{10\cos 2x} - \dfrac{3}{10\sqrt{5}}\tan^{-1}\sqrt{\dfrac{3\cos 2x - 5}{5}}$

1 1 $+ C$

[b] $\displaystyle\int \frac{9x^2}{\sqrt{7x - x^2}}\,dx$ 119 $a = \frac{7}{2}$ $\frac{1}{2}$

1 $= 9\left(-\dfrac{x + \frac{21}{2}}{2}\sqrt{7x - x^2} + \dfrac{3\left(\frac{49}{4}\right)}{2}\cos^{-1}\dfrac{\frac{7}{2} - x}{\frac{7}{2}}\right) + C$

$= -\dfrac{18x + 189}{2}\sqrt{7x - x^2} + \dfrac{1323}{8}\cos^{-1}\dfrac{7 - 2x}{7} + C$

$\frac{1}{2}$

You wish to estimate $\int_4^{10} f(x)\,dx$ using numerical approximation. Suppose that $f(4)=3$ and $f(10)=1$. **SCORE: _____ / 10 PTS**

Using 12 subintervals, the right hand sum gives 11.6, and the midpoint rule gives 11.9.

[a] Find the left hand sum using 12 subintervals. **Show how you got your answer.**

$\Delta x = \frac{10-4}{12} = \frac{1}{2}$

$L_{12} = R_{12} + f(4)\Delta x - f(10)\Delta x = 11.6 + 3(\frac{1}{2}) - 1(\frac{1}{2}) = 12.6$ (1) ($\frac{1}{2}$)

[b] Find the trapezoidal rule using 12 subintervals. **Show how you got your answer.**

$T_{12} = \frac{1}{2}(L_{12} + R_{12}) = \frac{1}{2}(12.6 + 11.6) = 12.1$ (1) ($\frac{1}{2}$)

[c] Based on your answers, would you guess that f is more likely to be concave up or down? Why?

CONCAVE UP SINCE $T_{12} > M_{12}$ ($\frac{1}{2}$) MUST HAVE BOTH CONCLUSION & REASON TO EARN ANY POINTS

[d] If $-2 \le f''(x) \le 1$ for $x \in [4, 10]$, find the best interval that $\int_4^{10} f(x)\,dx$ lies in based on the midpoint rule value above, and the error bounds formulae.

$|f''| \le 2$ ON $[4, 10] \longrightarrow E_{M_{12}} \le \frac{2(10-4)^3}{24(12)^2} = 0.125$ (3) ($\frac{1}{2}$)

$M_{12} - 0.125 \le \int_4^{10} f(x)\,dx \le M_{12} + 0.125$

(1) $11.775 \le \int_4^{10} f(x)\,dx \le 12.025$ (1)